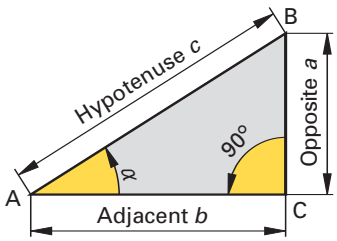
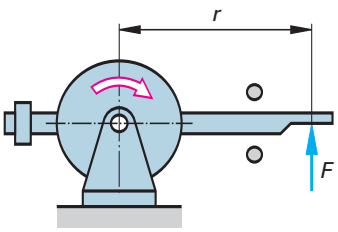
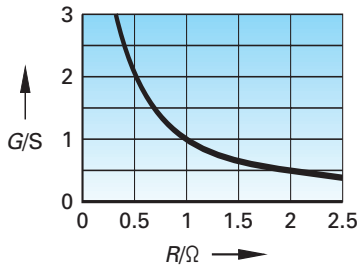
Section M: Mathematics, Physics, Theory of Circuits, Components

Mathematics 12

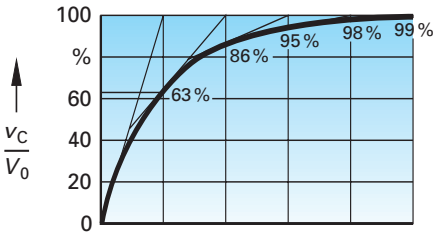


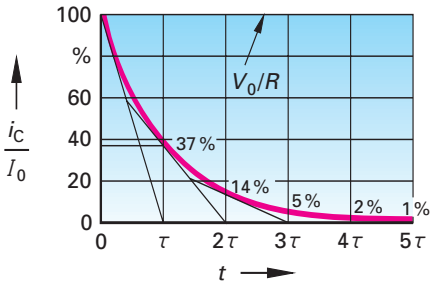
Physics 20



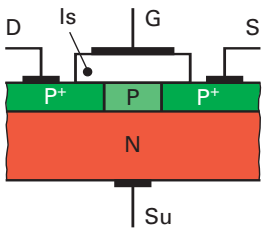


Circuit theory 36





Components 49



Symbols in This Book

Lower-case letters

Symbol	Meaning
a	acceleration
c	1. spec. heat capacity 2. electrochemical equivalent 3. propagation velocity of waves 4. general constant
d	1. diameter 2. distance 3. dissipation factor 4. duty cycle
e	elementary charge
f	1. frequency 2. filter factor
g	1. gravitational acceleration
h	height
i	time-controlled current
l	1. length 2. spacing
m	1. mass 2. number of strands
n	1. speed, number of revolutions 2. integer 1, 2, 3, ... 3. refractive index
o	overdrive factor
p	1. number of pole pairs 2. pressure 3. surface pressure
q	1. quantity 2. shunt current ratio
r	1. radius 2. rate 3. differential resistance
s	1. section, strength 2. normalized slip 3. correction
t	1. time 2. transformation ratio
v	1. velocity 2. time-controlled voltage
w	1. width 2. energy density 3. command variable
x	controlled variable
y	correcting variable
z	integer, e.g. number of teeth of a gear

Upper-case letters

Symbol	Meaning
A	1. area, 2. cross section 3. attenuation constant 4. apparent power 5. amplification factor
B	1. magn. flux density 2. current gain 3. number base 4. bandwidth
C	1. capacitance 2. thermal capacity 3. constant 4. coupling factor
D	1. electric flux density 2. damping factor 3. deflection coefficient
E	1. electric field strength 2. light density
F	1. force, 2. factor, 3. fault
G	1. conductance 2. amplification factor 3. gravitational force
H	magnetic field strength
I	1. electric current 2. light intensity
J	1. current density 2. mass moment of inertia
L	1. level 2. inductance
M	1. moment of force 2. memory capacity
N	number of turns
P	power, real or active power
Q	1. electric charge 2. heat 3. reactive power 4. resonant circuit quality
R	1. active resistance 2. spring rate 3. rigidity
S	1. susceptance 2. steepness 3. slip (absolute) 4. transmission quantity
T	1. cycle time 2. transmission factor 3. temperature in K 4. torque
V	1. voltage 2. volume
W	1. energy 2. work
X	reactance
Y	admittance
Z	1. impedance 2. wave impedance 3. oscillation impedance

Lower-case Greek letters

Symbol	Meaning
α (alpha)	1. angle 2. temperature coefficient 3. triggering angle
β (beta)	1. angle 2. short-circuit current amplification factor
γ (gamma)	1. angle 2. conductivity
δ (delta)	angle for losses
ε_0	electric field constant
ε (epsilon)	permittivity
ζ (zeta)	work ratio, utilisation ratio
η (eta)	efficiency
$\varkappa$ (kappa)	conductivity (optional symbol)
ϑ (theta)	temperature in °C
λ (lambda)	wavelength
μ (mu)	1. permeability 2. friction coefficient
μ_0	magnetic field constant
π (pi)	number 3.1415926...
ϱ (rho)	1. specific resistance 2. density
σ (sigma)	1. leakage factor 2. stress
τ (tau)	time constant
φ (phi)	angle, particularly phase-shift angle
ω (omega)	1. angular velocity 2. angular frequency

Upper-case Greek letters

Symbol	Meaning
Δ (Delta)	difference
Θ (Theta)	current linkage, (phase-shift angle in NA)
Φ (Phi)	1. magnetic flux 2. luminous flux
Ψ (Psi)	electric flux
Ω (Omega)	solid angle

Special symbols are created by adding one or more subscripts or other signs to the symbol.

Subscripts and Signs for Formula Symbols in This Book

Subscript, sign	Meaning
Numerals, symbols	
0	1. idle 2. vacuum 3. reference variable
1	1. input 2. order, sequence
2	1. output 2. order, sequence
3, 4, ...	order, sequence
$\hat{}$, e.g. $\hat{u}$	peak value
$\check{}$, e.g. $\check{u}$	minimum value
$\hat{\check{}}$, e.g. $\hat{\underset{\smile}{u}}$	1. peak-to-peak value 2. oscillation width
', e.g. u'	1. related to 2. note 3. derivation
△	delta connection
Y	star connection
Lower-case letters	
a	1. armature, 2. ambient, 3. actual
ab	absorbed
adm	admissible
del	delivered
amb	ambient air ...
b	1. bit 2. brake ...
c	1. cut-off, 2. crest, 3. comparison 4. centripetal...
d	1. referring to DC 2. duration, 3. digit 4. damping, 5. direction 6. desired 7. derivation, derived
e	1. exterior, 2. effective
eff	effective value
f	1. frequency 2. fall ...
h	high, upper
i	1. inner, internal 2. induced, 3. current 4. ideal, 5. intermediate 6. impulse
in	input
j	junction
k	kinetic
l	1. low, lower, 2. loss
m	1. magnetic 2. mean value 3. measuring, measured 4. modulated

Subscript	Meaning
max	maximum
min	minimum
n	1. nominal, 2. normal 3. noise
o	oscillator
off	switch off, turn off
out	output
p	1. parallel, 2. pause 3. pulse, 4. potential 5. pressure, 6. pre-
per	permissible
q	quality
r	1. reactive quantity 2. relative, related to 3. rise, 4. resonance 5. remanence, retentivity 6. reception
s	1. screen... 2. signal... 3. specific
sh	short circuit
st	step
t	1. test, 2. transverse 3. time
th	thermal, heat ...
tot	total
v	1. voltage, 2. visual
w	1. command variable 2. wind... 3. wave ...
x	1. unknown variable 2. in x-direction
y	1. correcting variable 2. in y-direction
z	zigzag connection
Upper-case letters	
A	1. ammeter 2. antenna 3. armature, 4. anode 5. acceleration, 6. area 7. amplifyer, amplifying
B	1. base 2. building
C	1. collector, 2. capacitive 3. clock pulse, 4. coercitive 5. cluster, 6. channel 7. maximum (max) contact voltage 8. carrier
D	1. drain, 2. data, 3. discharge
E	1. emitter 2. environment 3. earth

Subscript	Meaning
F	1. forward, 2. fault 3. (negative) feedback
G	gate
H	1. hysteresis 2. hall, 3. height 4. heat sink
K	cathode
L	1. inductive, 2. load 3. left, 4. Lorentz 5. loop
N	nominal, rated
O	1. operation, 2. operational earthing (network)
P	positive feedback
R	1. reverse, reward 2. active resistance 3. right 4. regular 5. red
S	1. source, 2. saddle ... 3. smoothing 4. switching 5. sluice ... 6. sector 7. system earthing 8. scanning
T	1. transformer ... 2. transverse 3. track, 4. test ...
V	voltage meter
W	weight
X	at the x-port
Y	1. at the y-port 2. star connection
Z	Zener ...
Lower-case Greek letters	
α (alpha)	in direction of the angle α
σ (sigma)	leakage
φ (phi)	phase-shift related
Upper-case Greek letters	
Δ (Delta)	referring to a difference

Subscripts may be combined, e.g. V_{CE} for collector-emitter voltage. Subscripts that consist of several letters may be reduced to the first letter.

Quantities and Units 1

Quantity	SI unit (other unit)	Unit symbol, unit equation
Length, area, volume, angle		
length	metre (sea mile) (inch)	m 1 sm = 1,852 m 1″ = 25.4 mm
area	square metre	m^2
volume	cubic metre (litre)	m^3 1 l = 1 dm^3 = = 1/1000 m^3
angle (plane)	radian (degree)	rad $1° = \frac{\pi}{180}$ rad
solid angle	steradian	sr
Time, frequency, velocity, acceleration		
time	second (minute) (hour) (day)	s 1 min = 60 s 1 h = 60 min = 3,600 s 1 d = 24 h
frequency	hertz	1Hz = 1/s
rotational speed-rotational frequency	per second (per minute)	1/s = 60/min
angular frequency	per second	1/s
velocity	metres per second (knot)	m/s 1 kn = 1 sm/h = 0.5144 m/s 1 km/h = $\frac{1}{3.6}$ m/s
angular velocity	radians per second	rad/s
acceleration	–	m/s^2
Mechanics		
mass	kilogram (carat) (tonne)	kg 1 Kt = 0.2 g 1 t = 1,000 kg
density	–	kg/m^3, kg/dm^3
moment of inertia	–	$kg \cdot m^2$
force	newton	1 N = 1 $kg \cdot m/s^2$
moment of force, torque	–	Nm
impulse	newton sec.	1 Ns = 1 $kg \cdot m/s$
pressure	pascal (bar)	1 Pa = 1 N/m^2 1 bar = 0.1 MPa = 10 N/cm^2
surface pressure, rigidity, modulus of elasticity	–	N/mm^2
work, energy	joule (electron volt)	1 J = 1 Nm = 1 Ws 1 eV = 0.1602 aJ
power	watt	1 W = 1 J/s = 1 Nm/s

Quantity	SI unit (other unit)	Unit symbol, unit equation
Electricity		
electric charge, electric flux	coulomb	1 C = 1 A · 1 s = 1 As
electric charge density, electric flux density	coulombs per square metre	C/m^2
space charge density	coulombs per cubic metre	C/m^3
electr. voltage, electr. potential	volt	1 V = 1 J/C
electr. field strength	volts per metre	1 V/m = 1 N/C
electr. capacitance	farad	1 F = 1 As/V = 1 C/V
current loading	amperes per metre	A/m
permittivity, dielectric constant	farads per metre	1 F/m = 1 C/(Vm)
electric current	ampere	1 A = 1 C/s
electric current density	amperes per m^2	A/m^2
electric resistance, active resistance, reactance, impedance	ohm	1 Ω = 1 V/A
electric conductance, susceptance, admittance	siemens	1 S = $\frac{1}{1\,\Omega}$
specific electric resistance	ohmmetre	1 Ωm = 100 Ωcm 1 $\Omega mm^2/m$ = 1μΩm
electric conductivity	siemens per metre	1 Sm/mm^2 = 1 MS/m
power	watt	1 W = 1 V · 1 A
reactive power	(var)	1 var = 1 V · 1 A
apparent power	(VA)	1 VA = 1 V · 1 A
inductance	henry	1 H = 1 Vs/A
work, energy	joule (watt-hour) (electron volt)	1 J = 1 Ws 1 Wh = 3.6 kNm 1 eV = 0.1602 aJ
Magnetism		
current linkage, magnetomotive force	ampere	A
magnetic field strength magnetisation	amperes per metre	A/m
magnetic flux	weber	1 Wb = 1 T · 1 m^2
magn. flux density magn. polarisation	tesla	1 T = 1 Wb/m^2 = 1 Vs/m^2
inductance	henry	1 H = 1 Vs/A
permeability	henrys per metre	1 H/m = 1 Vs/(Am)
magn. resistance	–	1/H = A/Vs

Quantities and Units 2

Quantity	SI unit (other unit)	Unit symbol, unit equation
Electromagnetic radiation (except light)		
radiant energy	joule	1 J = 1 Nm = 1 Ws
radiant power	watt	1 W = 1 J/s
radiant intensity	watt/sterad.	W/sr
radiant intensity	–	W/(sr · m^2)
irradiance	–	W/m^2
Light, optics		
light intensity	candela	cd
luminance	candelas per m^2	cd/m^2
luminous flux	lumen	lm
luminous efficacy	lumens per watt	lm/W 1 lx = 1 lm/m^2
light density	lux	
optical power of lenses	– (dioptre)	1/m 1 dpt = 1/m
Heat		
centigrade temperature	degree centigrade	°C
thermodynamic temperature	kelvin	K
temperature difference	kelvin	K
heat, inner energy	joule	1 J = 1 Ws
heat flow	watt	1 W = 1 J/s
thermal resistance (of components)	kelvins per watt	K/W
thermal conductivity	–	W/(K · m)
heat transfer coefficient	–	W/(K · m^2)
thermal capacity, entropy	joules per kelvin	J/K
specific thermal capacity	–	J/(kg · K)
Chemistry, molecular physics		
quantity of substance	mol(e)	mol
concentration of quantity of substance	–	mol/m^3
molar volume	–	m^3/mol
molality	–	mol/kg
molar mass	–	kg/mol
molar thermal capacity	–	J/(mol · K)
diffusion-coefficient	–	m^2/s

Quantity	SI unit (other unit)	Unit symbol, unit equation
Nuclear reaction, ionising radiation		
activity of a radioactive substance	bequerel	1 Bq = 1/s
absorbed dose	gray	1 Gy = 1 J/kg
absorbed dose rate	grays per second	Gy/s
dose equivalent	sievert	1 Sv = 1 J/kg
dose equivalent rate	sieverts per second	1 Sv/s = 1 J/(kg · s)
ion dose	coulombs per kilogram	C/kg
ion dose rate	amperes per kilogram	1 A/kg = 1 C/(kg · s)
Acoustics		
sound pressure	pascal	1 Pa = 1 N/m^2
sound particle velocity	metres per second	m/s
sound velocity (propagation velocity)	metres per second	m/s
volume velocity	–	1 m^3/s = 1 m^2 · 1 m/s
sound intensity	–	W/m^2
specific sound impedance	–	Pa · s/m
acoustic impedance	–	Pa · s/m^3
mechanical impedance	–	N · s/m
equivalent absorption area	square metre	m^2
Other disciplines		
distance in astronomy	(astronomical unit) parsec	1 AU = 149.6 Gm 1 pc = 30.857 Pm
mass in nuclear physics	(nuclear mass unit)	1 u = 1.66 · 10^{-27} kg
mass per unit length of textile fibres and threads	tex	1 tex = 1 g/kg
area of plots of land	are hectare	1 a = 100 m^2 1 ha = 100 a

Mathematical Symbols

Symbol	Meaning	Example
General symbols		
$\ldots n$	and so on until n	$k = 1, 2, 3, \ldots, n$
$\ldots$	and so on until infinity	$n = 1, 2, 3, \ldots$
		$\sqrt{2} = 1.41421\ldots$
Boolean algebra		
$\neg a, \overline{a}$	NOT a	$\overline{a \wedge b} = \neg (a \wedge b)$
$\wedge$	AND	$a \wedge b$ or $\wedge (a, b)$
$\vee$	OR	$a \vee b$ or $\vee (a, b)$
$\overline{\wedge}$	NOT AND (NAND)	$a \mathbin{\overline{\wedge}} b = \overline{a \wedge b}$
$\overline{\vee}$	NOT OR (NOR)	$a \mathbin{\overline{\vee}} b = \overline{a \vee b}$
Set theory		
$\in$	element of	$a \in M$: a is element of M
$\subset$	subset	$M_1 \subset M_2$: M_1 is subset of M_2
$\cup$	union of sets	$\{1, 2\} \cup \{3, 4\} = \{1, 2, 3, 4\}$
$\Rightarrow$	from this follows that	$a \cdot b = c \Rightarrow a = c/b$
Arithmetic		
$=$	equal to	$P = V \cdot I$
$\neq$	not equal, unequal	$4 \neq 5$
$\sim$	proportional	$u \sim r$
$\approx$	approximately	$\pi \approx 3.14$
$\mathrel{\hat{=}}$	corresponds to	1 cm $\mathrel{\hat{=}}$ 20 N
$<$	less than	$2 < 3$
$>$	greater than	$5 > 2$
$\leq$	less than or equal to	$a \leq 10$
$\geq$	greater than or equal to	$n \geq 7$
$\ll$	considerably less than	$R \ll 100\ \text{k}\Omega$
$\gg$	considerably greater than	$R_x \gg R_n$
$\cdot, \times$	times, multiplied	$a \cdot b = ab$, 12 x 3 = 36
$-, /, :$	divided by	$\frac{7}{2} = 7/2 = 7 : 2$
%	per cent	$1\,\% = 10^{-2}$, $50\,\% = 0.5$
‰	per thousand, per mil	$1‰ = 10^{-3}$, $8‰ = 0.8\,\%$
( ), [ ], { }	round, squared, curly, pointed brackets	$[a (b - c) + d]^2$
$\lvert z \rvert$	amount of z	$\lvert 4 \rvert = 4$, $\lvert -7 \rvert = 7$
$n!$	n factorial	$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$, $3! = 6$
Σ	sum	$\Sigma I = I_1 + I_2 + I_3 + \ldots$
Π	product	$\Pi k = k_1 \cdot k_2 \cdot k_3 \cdot \ldots$
$\sqrt{\ }$	square root of	$\sqrt{16} = 4$
$\sqrt[n]{\ }$	n^{th} root of	$\sqrt[3]{8} = 2$
π	pi	$\pi = 3.14159\ldots$

Symbol	Meaning	Example
∞	infinite	$n = 1, 2, 3, \ldots, \infty$
$\rightarrow$	versus, approaches, exceeds	$x \rightarrow a$, x approaches the value a
$f(x)$	function of x	$f(I) = I^2 \cdot R$
i or j	imaginary unit	$i^2 = j^2 = -1$
$\underline{Z}$	complex quantity Z	$\underline{Z} = R + jX$
Geometry, vectors		
$\parallel$	parallel	$g_1 \parallel g_2$, R1 $\parallel$ R2
$\uparrow\uparrow$	parallel in the same dir.	$g \uparrow\uparrow h$
$\uparrow\downarrow$	parallel in opposite dir.	$g_1 \uparrow\downarrow g_2$
$\perp$	orthogonal, perpendicular	$g \perp h$
$\triangle$	triangle	$\triangle$ ABC
$\cong$	congruent	$\triangle$ ABC $\cong \triangle$ DEF
$\sim$	similar	$\triangle P_1P_2P_3 \sim \triangle$ ABC
$\sphericalangle$	angle	$\sphericalangle$ ABC = $\sphericalangle$ (BA, BC), $\sphericalangle (\vec{a}, \vec{b})$
$\overline{AB}$	line segment AB	$\overline{P_1P_2}$
$\overset{\frown}{AB}$	arc AB	$\overset{\frown}{AB} = \sphericalangle\ \gamma$
$\vec{A}, \vec{B}$	vector A, vector B	$\vec{C} = \vec{A} + \vec{B}$
$\lvert\vec{A}\rvert$	absolute value of vector A	$\lvert\vec{F}\rvert = 50$ N
Differentiation, integration		
Δ	difference	$\Delta U = U_2 - U_1$
y'	y prime	y' is the first derivation of y, first derivative quotient $y' = dy/dx$
$\frac{dy}{dx}$	dy by dx or dy over dx	
$\int$	integral	$\int f(x)\,dx$, $\int_a^b f(x)\,dx$
Exponents, logarithms		
a^x	a to the power of x	5^3, 10^x
exp	exponential function	$\exp x = e^x$, with e = 2.718...
log	general logarithm	
$\log_a$	logarithm to the basis a	$\log_3 9 = 2$
lg	common logarithm	lg 2 = 0.30103...
lb	dyadic logarithm	lb 8 = 3
ln	natural logarithm	ln 10 = 2.3025...
Trigonometry		
sin	sine	$\sin\alpha$
cos	cosine	$\sin^2\alpha + \cos^2\alpha = (\sin\alpha)^2 + (\cos\alpha)^2 = 1$
tan	tangent	$\tan\alpha = \sin\alpha/\cos\alpha$
cot	cotangent	$\cot\alpha = 1/\tan\alpha$
arcsin	arc sine	$\sin\alpha = x \Rightarrow \arcsin x = \alpha$
arccos	arc cosine	$\cos\alpha = x \Rightarrow \arccos x = \alpha$
arctan	arc tangent	$\tan\alpha = x \Rightarrow \arctan x = \alpha$
arccot	arc cotangent	$\cot\alpha = x \Rightarrow \operatorname{arccot} x = \alpha$

Exponents, Unit Prefixes, Logarithms, Calculations According to the Rule of Three

Exponents

Values less than 1 can be expressed by multiples of decimal powers with negative exponents.
Values greater than 1 can be expressed by multiples of decimal powers with positive exponents.

Value	0.001	0.01	0.1	1	10	100	1,000	10,000	100,000	1,000,000
Decimal powers	10^{-3}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}	10^{3}	10^{4}	10^{5}	10^{6}

Powers of two are used in digital engineering. The base here is 2.

Value	1/128	1/64	1/32	1/16	1/8	1/4	1/2	1	2	4	8	16	32	64	128
Powers of two	2^{-7}	2^{-6}	2^{-5}	2^{-4}	2^{-3}	2^{-2}	2^{-1}	2^{0}	2^{1}	2^{2}	2^{3}	2^{4}	2^{5}	2^{6}	2^{7}

Unit prefixes

Used with physical quantities (also transfer rates)

Prefix symbol	Prefix	Meaning (factor)
a	atto	10^{-18}
f	femto	10^{-15}
p	pico	10^{-12}
n	nano	10^{-9}
µ	micro	10^{-6}
m	milli	10^{-3}
c	centi	10^{-2}
d	deci	10^{-1}

Prefix symbol	Prefix	Meaning (factor)
da	deca	10
h	hecto	10^{2}
k	kilo	10^{3}
M	mega	10^{6}
G	giga	10^{9}
T	tera	10^{12}
P	peta	10^{15}
E	exa	10^{18}

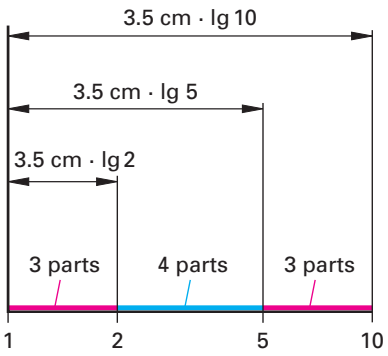
Used with memory sizes specified in bits or bytes

Prefix symbol	Prefix	Meaning (factor)
–	–	–
–	–	–
K	kilo	2^{10}
M	mega	2^{20}
G	giga	2^{30}
T	tera	2^{40}
P	peta	2^{50}
E	exa	2^{60}

For large mass storage units, often the meaning of the physical quantities applies.

Prefixes may not be combined. You can assign only one prefix per unit.

Logarithms



Logarithmic sectioning

The logarithm (log) indicates to which power a base has to be raised in order to obtain the logarithm argument. The following applies

$a^b = c,\ \log_a c = b$

The common logarithm (lg) has the base 10. The natural logarithm (ln) has the base of the e-function (e=2.718...). The dyadic logarithm (lb) has the base 2.

Extensive number ranges can be represented in a more structured way when using a logarithmic scale.

$$\log_a c = \frac{c}{\ln a} = \frac{c}{\lg a}$$

$$\log_a(cd) = \log_a c + \log_a d$$

$$\log_a \frac{c}{d} = \log_a c - \log_a d$$

$$\log_a(c^m) = m \cdot \log_a c$$

$$\log_a \sqrt[n]{c} = \frac{1}{n}\log_a c$$

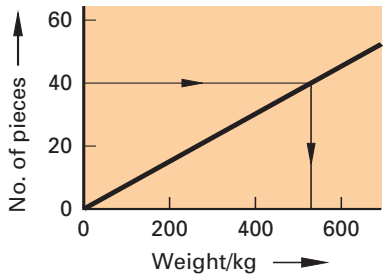
$$\lg x = \ln x / \ln 10$$

$$\ln x = \lg x / \lg e$$

$$\operatorname{lb} x = \ln x / \ln 2$$

$$\operatorname{lb} x = \lg x / \lg 2$$

Calculation according to the rule of three



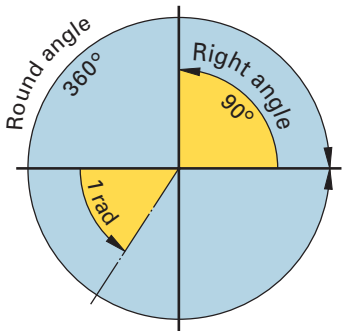
Calculation acc. to the rule of three of a proportional relation

Steps of approach	Example
Proportional relation (unit obtained by division)	
1. Statement 2. Calculation for 1 object 3. Calculation for *z* objects	*n* elements have a weight of *a* kg 1 element has a weight of *a*/*n* kg *z* elements have a weight of *z* · *a*/*n* kg
Inverted proportional relation (unit obtained by multiplication)	
1. Statement 2. Calculation for 1 object 3. Calculation for z objects	*n* workers need *a* hours 1 worker needs *n* · *a* hours *z* workers need *n* · *a*/*z* hours

Angles, Trigonometric Functions, Percentage Calculation

Figures	Explications	Notes, formulas

Angles



Angle dimensions

The units referring to angles are degree, centesimal degree and radian.

The *round angle* has

a) 360 ° (degrees)
b) 400 gon (centesimal degrees)
c) 2π rad (radian)

The unit radian corresponds to the proportion of the circular arc length to the radius in a circle.

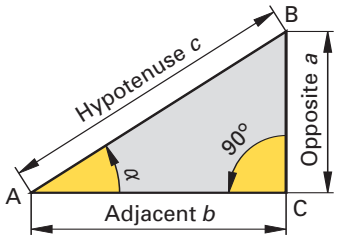
$$\alpha_r = \alpha° \cdot \frac{\pi}{180°}$$

$$1\ \text{rad} = \frac{360°}{2\pi} = 57.296°$$

Important angles

Round angle	Straight angle	Right angle
360°	180°	90°
$2 \cdot \pi$ rad	π rad	$\frac{\pi}{2}$ rad
400 gon	200 gon	100 gon

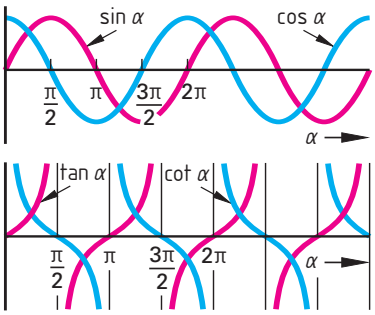
Angle functions



Right-angled triangle

The longest side (*c*) of the right-angled triangle is referred to as the *hypotenuse*. It is the side opposite the right angle. The two other sides (*a* and *b*) of the triangle form the right angle. These sides are referred to as the *catheti* or simply *legs* of the triangle. The leg (*a*) opposite the acute angle α is the *opposite*. The leg contiguous to the angle α is the adjacent (*b*).

An angle in a right-angled triangle can be defined by its angle degrees or as a *ratio of two triangle sides*. The ratio of the sides depends on the size of the angle. That is why the ratios of two sides in a right-angled triangle are referred to as *angle functions* (function = dependence) or trigonometric functions.



Trigonometric functions

$$\text{Sine} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{Cosine} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{Tangent} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\text{Cotangent} = \frac{\text{adjacent}}{\text{opposite}}$$

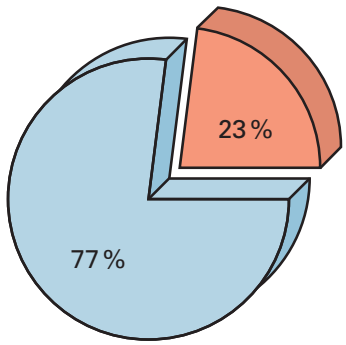
$$\sin\alpha = \frac{a}{c}$$

$$\cos\alpha = \frac{b}{c}$$

$$\tan\alpha = \frac{a}{b}$$

$$\cot\alpha = \frac{b}{a}$$

Percentage calculation



Per cent (pro cent in Latin) means "per hundred". The total quantity (basic quantity) is always equal to one hundred, the partial quantity (percentage) is expressed in per cent (= hundredths).

23% of 300 € is equal to **69 €**

percentage | basic value | percentage amount

$$\text{percentage} = \frac{100 \cdot \text{percentage amount}}{\text{basic value}}$$

Percentage calculation

$$p = \frac{P \cdot 100\%}{B}$$

Calculation of interest

$$I = \frac{C_0 \cdot p \cdot n}{100\%}$$

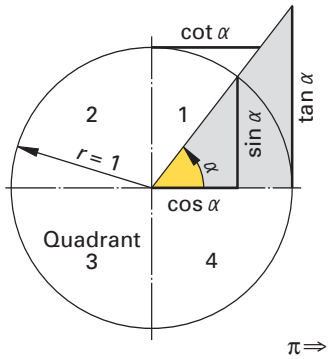
Calculation of compound interest

$$C_n = C_0 \cdot \left(1 + \frac{p}{100\%}\right)^n$$

a, b, c	legs of a right-angled triangle	I	interest per year	α, β, γ	angles in a triangle
B	basic amount	n	term in years	$\alpha°$	degrees of an angle
C_0	starting capital	P	percentage amount	α_r	radian of an angle
C_n	capital after n years	p	percentage in %, interest rate in %		

Relationships Between Trigonometric Functions

Angular relationships in a right-angled triangle



According to the Pythagorean theorem:

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

Because $\tan \alpha = \dfrac{\text{opposite}}{\text{adjacent}}$

and $\cot \alpha = \dfrac{\text{adjacent}}{\text{opposite}}$

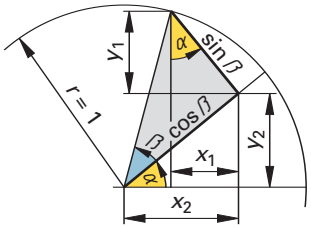
$$\pi \Rightarrow \quad \tan \alpha \cdot \cot \alpha = 1$$

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \frac{1}{\sqrt{1 + \cot^2 \alpha}}$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \frac{1}{\sqrt{1 + \tan^2 \alpha}}$$

The root is either positive or negative depending on the quadrant where the angle is located.

Quadrant	1	2	3	4
sin	+	+	–	–
cos	+	–	–	+



$$x_1 = \sin \alpha \cdot \sin \beta$$
$$x_2 = \cos \alpha \cdot \cos \beta$$
$$y_1 = \cos \alpha \cdot \sin \beta$$
$$y_2 = \sin \alpha \cdot \cos \beta$$
$$\sin (\alpha + \beta) = y_1 + y_2$$
$$\cos (\alpha + \beta) = x_2 - x_1$$

$$\sin (\alpha + \beta) \cdot \sin (\alpha - \beta) = \cos^2 \beta - \cos^2 \alpha$$

$$\sin (\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$
$$\cos (\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$
$$\tan (\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$
$$\cot (\alpha \pm \beta) = \frac{\cot \alpha \cot \beta \mp 1}{\cot \beta \pm \cot \alpha}$$

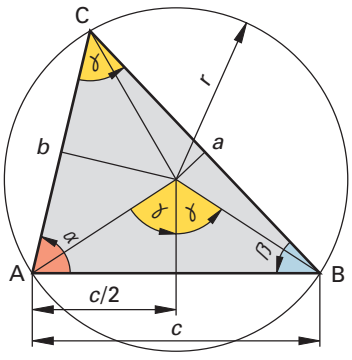
$$\cos (\alpha + \beta) \cdot \cos (\alpha - \beta) = \cos^2 \beta - \sin^2 \alpha$$

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{\alpha \pm \beta}{2} \cos \frac{\alpha \mp \beta}{2}$$
$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$
$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$
$$\tan \alpha \pm \tan \beta = \frac{\sin (\alpha \pm \beta)}{\cos \alpha \cos \beta}$$
$$\cot \alpha \pm \cot \beta = \pm \frac{\sin (\alpha \pm \beta)}{\sin \alpha \sin \beta}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$
$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$
$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{2}{\cot \alpha - \tan \alpha}$$
$$\cot 2\alpha = \frac{\cot^2 \alpha - 1}{2 \cot \alpha} = \frac{\cot \alpha - \tan \alpha}{2}$$

Angular relationships in a general triangle

Law of sines



$$\frac{c}{2} = r \cdot \sin \gamma$$
$$\frac{a}{2} = r \cdot \sin \alpha$$
$$\frac{b}{2} = r \cdot \sin \beta$$

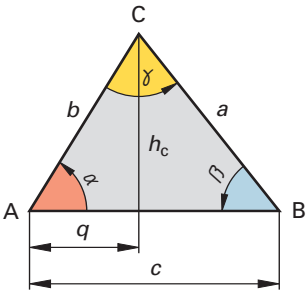
$$c = 2r \cdot \sin \gamma,\ a = 2r \cdot \sin \alpha,\ b = 2r \cdot \sin \beta$$

$$a : b : c = \sin \alpha : \sin \beta : \sin \gamma$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

Two sides of a triangle always behave like the sine values of the opposite angles.

Law of cosines



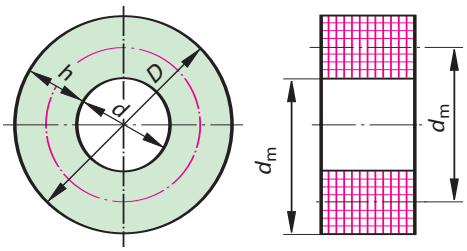
$$h_c = b \cdot \sin \alpha$$
$$q = b \cdot \cos \alpha$$
$$a^2 = h_c^2 + (c - q)^2$$
$$a^2 = b^2 \sin^2 \alpha + b^2 \cos^2 + c^2 - 2bc \cdot \cos \alpha$$
$$a^2 = b^2 + c^2 - 2bc \cdot \cos \alpha$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos \alpha$$
$$b^2 = a^2 + c^2 - 2ac \cdot \cos \beta$$
$$c^2 = a^2 + b^2 - 2ab \cdot \cos \gamma$$

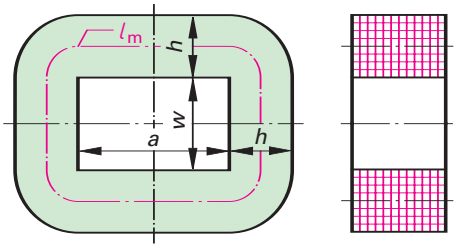
In a triangle, the square of one side length is equal to the sum of the squares of the remaining side lengths minus two times the product of these side lengths and the cosine of the enclosed angle.

Lengths and Areas

Wire lengths



Round coils



Rectangular coils

Round coils

$l = l_m \cdot N$

$l_m = \pi \cdot d_m$

$$l = \pi \cdot d_m \cdot N$$

$$h = \frac{D - d}{2}$$

$d_m = d + h$

or

$$d_m = \frac{D + d}{2}$$

$d_m = D - h$

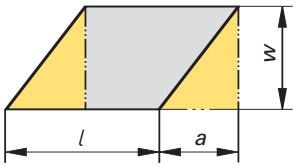
Rectangular coils

$l = l_m \cdot N$

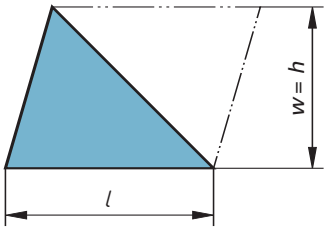
$l_m = 2a + 2b + \pi \cdot h$

$$l = (2a + 2b + \pi \cdot h) \cdot N$$

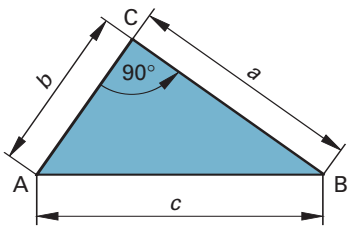
Areas



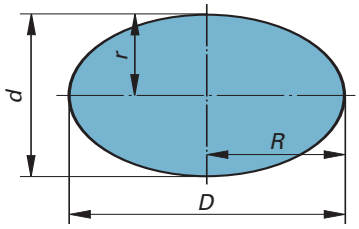
Parallelogram (rhomboid)



General triangle



Right-angled triangle



Ellipse

Square

$$A = s^2$$

$$p = 4 \cdot s$$

$e = \sqrt{2} \cdot s = 1.41 \cdot s$

Rectangle

$$A = l \cdot w$$

$$p = 2 \cdot (l + w)$$

$e = \sqrt{l^2 + w^2}$

Parallelogram

$$A = l \cdot w$$

$p = 2 \cdot (l + \sqrt{l^2 + w^2})$

General triangle

$$A = \frac{l \cdot w}{2}$$

Right-angled triangle

Pythagorean theorem:

$$c^2 = a^2 + b^2$$

$p = a + b + \sqrt{a^2 + b^2}$

In a right-angled triangle, the right angle is enclosed by the two catheti. The hypotenuse is opposite to it.

Circle

$$p = \pi \cdot d$$

$p = 2 \cdot \pi \cdot r$

$$A = \pi \cdot r^2$$

$$A = \frac{\pi \cdot d^2}{4}$$

$A \approx 0.785 \cdot d^2$

Ellipse

$$p \approx \pi \cdot \frac{D + d}{2}$$

$$A = \pi \cdot R \cdot r$$

$$A = \frac{\pi \cdot D \cdot d}{4}$$

A	area	D	(great) diameter	h	height	p	perimeter, sum of side lengths
a	side	d	(small) diameter	l	length	r, R	radius
w	width	d_m	medium diameter	l_m	mean length	s	side length
c	side (hypotenuse)	e	width across corners, diagonal	N	number of turns		